TEST KALEIDOSCOPE (MODULAR ARITHMETIC),
October 14th, 2019, 3:00pm–4.00pm,
Aletta Jacobshal 01.

Please provide complete arguments for each of your answers The exam consists of 3 questions. You can score up to 6 points for each question, and you obtain 2 points for free

In this way you will score in total between 2 and 20 points

(1) In the eastern part of the United States one finds rather annoying bugs called 'cicadas'. In the states Maryland and North Carolina, two kinds of cicadas are common: the so-called "East Coast Brood" (ECB's), and the "Great Southern Brood" (GSB's) Fortunately cicadas live mostly underground However, periodically they emerge in huge numbers, with a serious impact on life in general. For the ECB's this happens once every 17 years in mid-May and their most recent mass emergence took place in 2013. Mass emergence of GSB's happens once every 13 years in mid-May, most recently in 2011.
   (a) (1 point) Show that a mass emergence of ECB's happens in year N precisely when $N \equiv 2013 \bmod 17$.
   (b) (2 points.) The most disastrous years are clearly the ones in which ECB's and GSB's emerge together. Determine whether this ever happens between the years 1900 and 2100.

(2) Consider the integer $n = 3^{14102019} + 3$
   (a) (1 point.) Show that $10|n$.
   (b) (2 points.) Is n a unit modulo 7?

(3) In this final exercise p and q are arbitrary prime numbers, and $p \neq q$.
   (a) (1 point) How many integers n exist with $1 \leq n \leq p^2q$ and $(pq)|n$?
   (b) (2 points.) Determine the number of elements in $(\mathbb{Z}/p^2q\mathbb{Z})^\times$, i.e., the number of elements in $\mathbb{Z}/p^2q\mathbb{Z}$ that have an inverse.